UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MEK4540/9540 – Composite Materials and Structures
Date of examination:	Thursday 05-06-2008
Examination hours:	1430 – 1730
This examination paper consists of 3 pages	
Appendix:	Useful formulae (1 page)
Permitted aids:	Rottmann's formula compilation + approved calculator

Make sure that your copy of this examination paper is complete before answering.

Problem 1 (20%)

a) Name a commonly used thermoplastic resin, a thermoset resin and a reinforcing fibre material and give typical values for stiffness and strength.

b) Explain the meaning of *glass transition temperature* (Tg) and sketch the elastic modulus as a function of the temperature for an amorphous thermoplastic, a semi-crystalline thermoplastic and a thermoset.

c) Explain the most important principles of RTM and pultrusion and illustrate your answer with sketches.

d) Explain the meaning of *load transfer length* (l_t) and *critical fibre length* (l_c). Illustrate with sketches.

Problem 2 (40%)

A laminate consists of plies with unidirectional fibres in a thermoset matrix and has the following elastic properties:

$$E_L = 130\ 000\ \text{MPa}$$
$$E_T = 9000\ \text{MPa}$$
$$G_{LT} = 5000\ \text{MPa}$$
$$\nu_{LT} = 0.25$$

a) Determine the compliance matrix S and the stiffness matrix Q in the LT-system for the given plies.

b) Calculate the stiffness matrix for a ply which has fibres oriented at 90° with respect to the global x-axis.

c) The laminate has the configuration [0/90/90/0] with respect to the global x-axis and is exposed to an in-plane loading. The following strains are measured with strain gauges in the global xy-coordinate system:

$$\varepsilon_x = 2000 \times 10^{-6}$$
$$\varepsilon_y = 500 \times 10^{-6}$$
$$\gamma_{xy} = 1000 \times 10^{-6}$$

Calculate the in-plane loading (N_x, N_y, N_{xy}) if each ply has thickness 1 mm.

d) Calculate the elastic modulus in the global x-direction

e) The following rupture strains have been measured in uniaxial tensile tests for individual plies:

$$\varepsilon_{LU} = 0.01$$
$$\varepsilon_{TU} = 0.005$$
$$\gamma_{LTU} = 0.05$$

In which ply or plies will failure occur first for uniaxial loading in the global x-direction, and what type of failure is this?

f) Calculate the load at the first ply failure in part e) above.

Problem 3 (40%)

PART A

a) Which advantages does a sandwich structure have compared with a conventional beam or plate structure?

b) Which failure mechanisms should be considered when a sandwich beam is to be designed to withstand a given loading? Both lateral and axial laods are to be taken into account.

PART B

a) Define the quantities *bending stiffness*, D, and *shear stiffness*, S, both per unit width, for a sandwich beam.

b) Figure 1 shows a simply supported, horizontal sandwich beam, AB, with length L. a vertical, uniformly distributed load q per unit area is applied over the entire length of the beam. Both face sheets can be considered as thin and the core as weak (compliant).

By use of partial deflections, or otherwise, find an expression for the vertical displacement $w(x)$ og and show that the displacement δ at the centre of the beam is given by

$$\delta = \frac{5qL^4}{384D}\left(1 + \frac{48D}{5SL^2}\right)$$

c) Investigate and discuss how the solution in item b) above changes when the ratio D/SL^2 is increased from zero to a large value.

d) Describe briefly how the analysis must be modified if end A is built-in while B remains simply supported.

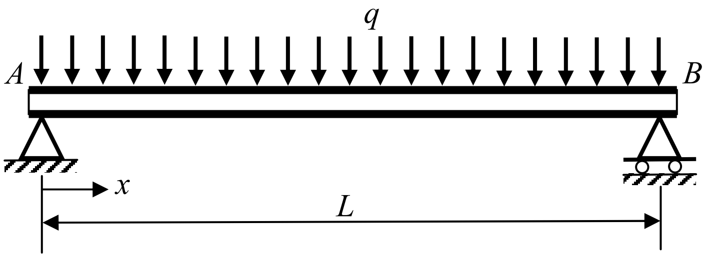
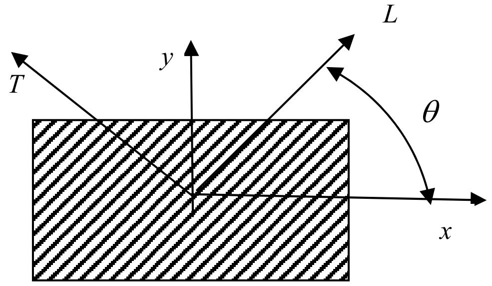


Figure 1: Simply supported sandwich beam with uniformly distributed load.

USEFUL FORMULAE

$$[T_1] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}$$



$$[T_2] = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix}$$

$c = \cos\theta$

$s = \sin\theta$

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \frac{1}{2}\gamma_{LT} \end{bmatrix} = [T_1] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix}$$ (tensor strains) or $$\begin{bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{bmatrix} = [T_2] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$ (“engineering strains”)

$$\begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix} = [T_1] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$[\overline{Q}] = [T_1]^{-1}[Q][T_1]$ (tensor strains) or $[\overline{Q}] = [T_1]^{-1}[Q][T_2]$ (“engineering strains”)

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k - h_{k-1}\right)$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^3 - h_{k-1}^3\right)$$